

# EE3209 - Communications Laboratory

## Lab Manual

California State University, Los Angeles  
College of Engineering, Computer Science, and Technology  
Department of Electrical and Computer Engineering



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# Lab 4: Image Rejection

## Objective:

This laboratory exercise illustrates the image problem in superheterodyne receivers. Image rejection is carried out using complex filtering. This lab introduces a processing technique that is straightforward in a software-defined radio, but is virtually unavailable in a conventional hardware-based radio. The purpose of this exercise is:

- Provide additional practice in programming the USRP
- Introduce the concept of image rejection in superheterodyne receiver with AM
- Explore the concept of complex filtering in the receiver

## Background:

### Frequency Conversion and Heterodyne Receivers

As we saw and discussed in Lab 2, a tunable BPF must be installed before the detector stage to recover information from our desired station. Figure 1 shows a simple architecture of coherent receiver we studied in Lab 2.

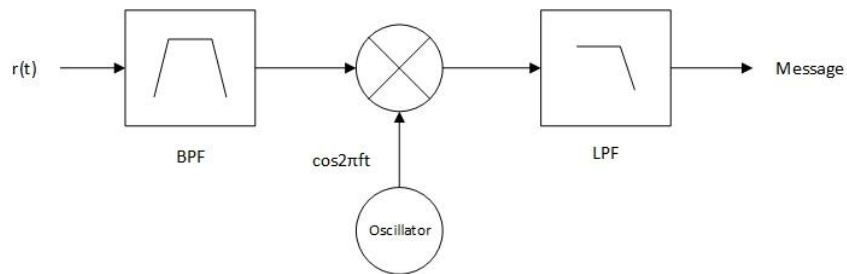


Figure 1: Coherent Receiver Architecture

The carrier frequency we used in Lab 2 was  $f_c = 915.1 \text{ MHz}$  while the signal bandwidth was  $BW = 10 \text{ kHz}$ . In this case, the **Quality Factor** of the filter which is also known as **Q-Factor** of the filter is:

$$Q \triangleq \frac{f_c}{BW} = \frac{915.1 \text{ MHz}}{10 \text{ kHz}} = 91,510$$

Eq. 1



The problem is that  $Q \gg 10$  is extremely hard to achieve and in our case it's impossible to have such a big  $Q$ . The solution is to shift the RF signal to a lower frequency where it is much easier to implement and construct a BPF. The process of shifting the RF frequency to a lower frequency in which a BPF can be implemented to reject all other unwanted signals is called **Heterodyning** [1]. In today's most communication receivers, the received signal at carrier frequency  $f_c$  is downconverted to a signal at **intermediate** frequency  $f_{IF}$  for amplification and filtering prior to demodulation.

In the USRP, the frequency conversion can be carried out by offsetting the receiver's carrier frequency from the transmitter's carrier frequency. To avoid confusion, we define three different frequencies,  $f_c$ ,  $f_{LO}$  and  $f_{IF}$ . In Lab 2, we set the transmitter's carrier frequency to  $f_c = 915.1 \text{ MHz}$ . The receiver's carrier oscillator is usually referred to as a **Local** oscillator, and its frequency is referred to as the **Local Oscillator** frequency,  $f_{LO}$ . In Lab 2, we set the receiver's local oscillator frequency to  $f_{LO} = 915 \text{ MHz}$ , these settings provide an intermediate frequency  $f_{IF} = f_c - f_{LO} = 100 \text{ kHz}$ .

In the USRP receiver, frequency conversion is carried out in hardware by multiplying the received signal by  $\cos(2\pi f_{LO}t)$  and  $-\sin(2\pi f_{LO}t)$ . For example, suppose the received signal is the AM waveform:

$$r(t) = A \left[ 1 + \mu \frac{m(t)}{m_p(t)} \right] \cos(2\pi f_c t + \theta)$$

Eq. 2

The receiver forms the two terms in Eq. 3:

$$\begin{aligned} r(t) \times \cos(2\pi f_{LO}t) &= \underbrace{A \left[ 1 + \mu \frac{m(t)}{m_p(t)} \right] \cos(2\pi f_c t + \theta)}_{r(t)} \times \cos(2\pi f_{LO}t) \\ &= \frac{1}{2} A \left[ 1 + \mu \frac{m(t)}{m_p(t)} \right] \cos(2\pi(f_c - f_{LO})t + \theta) + \frac{1}{2} A \left[ 1 + \mu \frac{m(t)}{m_p(t)} \right] \cos(2\pi(f_c + f_{LO})t + \theta) \end{aligned}$$

Eq. 3

Note the first term in Eq. 3 has the intermediate frequency at  $f_{IF} = f_c - f_{LO} = 100 \text{ kHz}$ , while the second term has at a much higher frequency  $f_{IF} = f_c + f_{LO} = 1830.1 \text{ MHz}$ . The higher-frequency term is removed by filtering in the USRP, providing the **in-phase** signal:



$$r_I(t) = A_r \left[ 1 + \mu \frac{m(t)}{m_p(t)} \right] \cos(2\pi f_{IF}t + \theta)$$

Eq. 4

where  $A_r = \frac{1}{2}A$  and  $f_{IF} = f_c - f_{LO}$ . The receiver also forms a second signal, **quadrature** part:

$$\begin{aligned} -r(t) \times \sin(2\pi f_{LO}t) &= -A \underbrace{\left[ 1 + \mu \frac{m(t)}{m_p(t)} \right] \cos(2\pi f_c t + \theta)}_{r(t)} \times \sin(2\pi f_{LO}t) \\ &= \frac{1}{2}A \left[ 1 + \mu \frac{m(t)}{m_p(t)} \right] \sin(2\pi(f_c - f_{LO})t + \theta) - \frac{1}{2}A \left[ 1 + \mu \frac{m(t)}{m_p(t)} \right] \sin(2\pi(f_c + f_{LO})t + \theta) \end{aligned}$$

Eq. 5

Again, the high-frequency term is removed, and the receiver provides the quadrature signal:

$$r_Q(t) = A_r \left[ 1 + \mu \frac{m(t)}{m_p(t)} \right] \sin(2\pi f_{IF}t + \theta)$$

Eq. 6

where  $A_r = \frac{1}{2}A$  and  $f_{IF} = f_c - f_{LO}$ . A conventional hardware-based receiver normally works with the in-phase signal given by Eq. 4. The USRP combines the in-phase and quadrature signals to form the complex IF signal  $\tilde{r}(t)$  given by:

$$\begin{aligned} \tilde{r}(t) &= r_I(t) + jr_Q(t) \\ &= A_r \left[ 1 + \mu \frac{m(t)}{m_p(t)} \right] \cos(2\pi f_{IF}t + \theta) + j \left( A_r \left[ 1 + \mu \frac{m(t)}{m_p(t)} \right] \sin(2\pi f_{IF}t + \theta) \right) \\ &\Rightarrow \tilde{r}(t) = A_r \left[ 1 + \mu \frac{m(t)}{m_p(t)} \right] e^{j(2\pi f_{IF}t + \theta)} \end{aligned}$$

Eq. 7

This complex signal is what is actually provided to the user by the **Fetch Rx Data VI**, that's the reason we used the **Complex to Re/Im** block after the BPF to extract the real part of the signal and do the demodulation process.

The spectrum  $R(f)$  of the received signal  $r(t)$  given in Eq. 2 is shown in Figure 2. Since  $r(t)$  is a real-valued signal, its spectrum contains both positive and negative-frequency components.



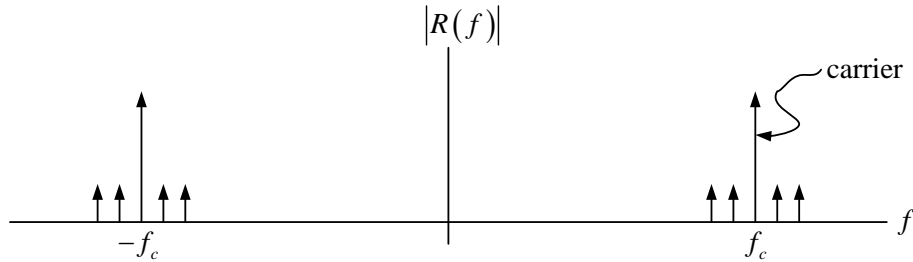


Figure 2: Spectrum of Received AM Signal at USRP

After frequency conversion, the complex IF signal  $\tilde{r}(t)$  has the spectrum  $\tilde{R}(f)$  shown in Figure 3:

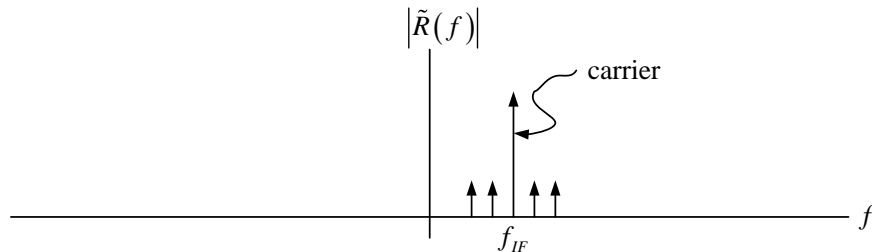


Figure 3: Spectrum of Complex IF signal

In Lab 2, we passed the complex IF signal through a Chebyshev BPF with high and low cut-off frequencies of 105 kHz and 95 kHz respectively. The frequency response of this BPF is shown in Figure 4. It is intended that signals at carrier frequencies other than  $f_{IF}$  will be rejected by the filter.

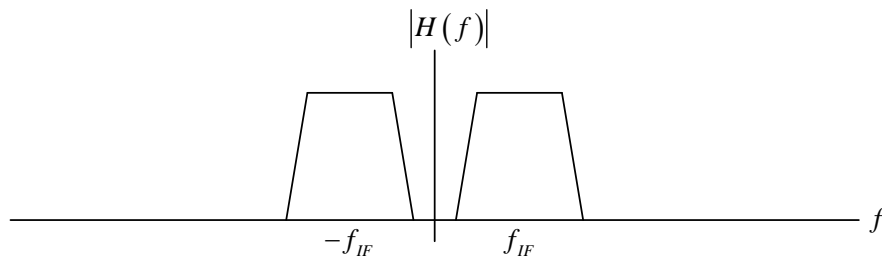


Figure 4: Frequency Response of Intermediate-Frequency Filter

## Image Signal

Suppose that there is a second signal received along with the signal of Eq. 2, and that this signal is given by Eq. 8:

$$r_2(t) = A_2 \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] \cos(2\pi f_{IM} t + \theta)$$

Eq. 8

Where image frequency is:

$$f_{IM} = f_{LO} - f_{IF}$$

Eq. 9

If we carry out the analysis of frequency conversion in the receiver for the second signal, we have:

$$\begin{aligned} r_2(t) \times \cos(2\pi f_{LO}t) &= \underbrace{A_2 \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] \cos(2\pi f_{IM}t + \theta_2)}_{r_2(t)} \times \cos(2\pi f_{LO}t) \\ &= \frac{1}{2} A_2 \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] \cos(2\pi(f_{IM} - f_{LO})t + \theta_2) + \frac{1}{2} A_2 \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] \cos(2\pi(f_{IM} + f_{LO})t + \theta_2) \end{aligned}$$

Using Eq. 9:

$$= \frac{1}{2} A_2 \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] \cos(2\pi(-f_{IF})t + \theta_2) + \frac{1}{2} A_2 \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] \cos(2\pi(2f_{LO} - f_{IF})t + \theta_2)$$

Eq. 10

Since the second term has a much higher frequency, it is removed by filtering in the USRP, providing the in-phase signal:

$$r_{2I}(t) = A_{r2} \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] \cos(2\pi(-f_{IF})t + \theta_2)$$

Eq. 11

where  $A_{r2} = \frac{1}{2} A_2$ . If we carry out the analysis of quadrature part for the second signal, we have:

$$\begin{aligned} -r_2(t) \times \sin(2\pi f_{LO}t) &= -\underbrace{A_2 \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] \cos(2\pi f_{IM}t + \theta_2)}_{r_2(t)} \times \sin(2\pi f_{LO}t) \\ &= \frac{1}{2} A_2 \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] \sin(2\pi(f_{IM} - f_{LO})t + \theta_2) - \frac{1}{2} A_2 \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] \sin(2\pi(f_{IM} + f_{LO})t + \theta_2) \end{aligned}$$

Using Eq. 9:

$$= \frac{1}{2} A_2 \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] \sin(2\pi(-f_{IF})t + \theta_2) - \frac{1}{2} A_2 \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] \sin(2\pi(2f_{LO} - f_{IF})t + \theta_2)$$

Eq. 12

Since the second term has a much higher frequency, it is removed by filtering in the USRP, providing the quadrature signal:



$$r_{2Q}(t) = A_{r2} \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] \sin(2\pi(-f_{IF})t + \theta_2)$$

Eq. 13

where  $A_{r2} = \frac{1}{2}A_2$ . The USRP combines the in-phase and quadrature signals to form the complex IF signal  $\tilde{r}_2(t)$  given by:

$$\tilde{r}_2(t) = A_{r2} \left[ 1 + \mu_2 \frac{m_2(t)}{m_{p2}(t)} \right] e^{j(2\pi(-f_{IF})t + \theta)}$$

Eq. 14

The spectrum  $\tilde{R}_2(f)$  of the signal is shown in Figure 5:

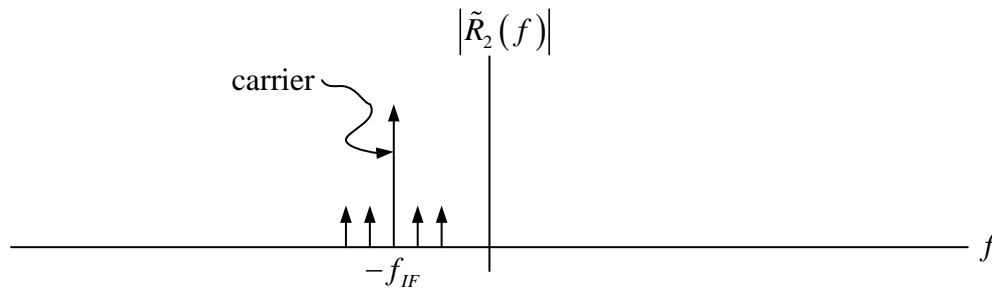


Figure 5: Spectrum of the Image Signal

The signals  $r(t)$  and  $r_2(t)$  are said to be **images** of one another. A glance at the frequency response shown in Figure 4 shows that both  $r(t)$  and  $r_2(t)$  will pass through the IF filter, and the two signals will interfere with each other in the demodulator that follows the IF filter. The relationship between the frequencies of the two image signals is worth noting. One signal,  $r(t)$ , is at a carrier frequency of  $f_c = f_{LO} + f_{IF}$ , while the other signal,  $r_2(t)$ , is at a carrier frequency of  $f_{IM} = f_{LO} - f_{IF}$ . These carrier frequencies are symmetrically arranged about the receiver's frequency  $f_{LO}$ , the way a physical object and its image are symmetrically distant from the surface of a mirror.

## Image Rejection

In a conventional receiver, the unwanted image is normally removed before the frequency conversion step. This requires placing an analog filter in the receiver front end. Clearly, we will not have the option of adding analog hardware to the USRP, so we will take advantage of some clever signal processing to remove the

unwanted image signal after frequency conversion. Notice in Figure 3 that the desired signal shows up at the intermediate frequency  $f_{IF}$ , while in Figure 5 we see that the unwanted image signal shows up at the intermediate frequency  $-f_{IF}$ . A conventional BPF passes both of these signals, but if we can construct a filter with a positive-frequency-only passband, we will be able to pass the desired signal while rejecting the unwanted image. Figure 6 shows the filter frequency response we have in mind.

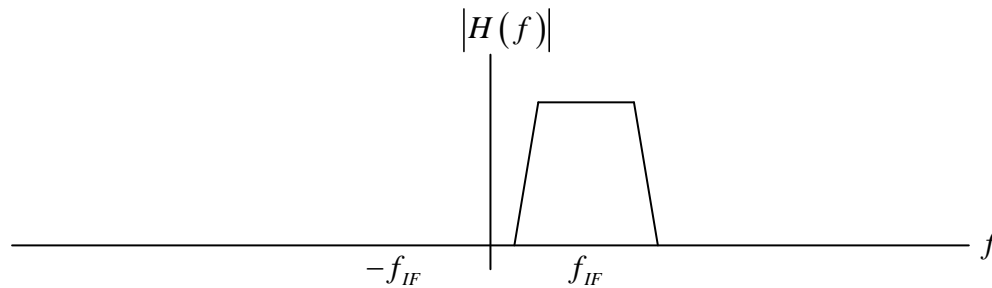


Figure 6: frequency Response of a Complex Image-Rejection Filter

A frequency response such as that shown in Figure 6 lacks symmetry about the vertical axis, which implies that the impulse response of the filter is complex-valued. A filter with a complex-valued impulse response is difficult to produce in hardware, but, as we will see, it is easily produced in software.

On a practical note, an analog image rejection filter must be tunable if the receiver is to be capable of receiving signals at a range of carrier frequencies  $f_c$ . To make the filter tunable, it usually must be kept simple, which constrains the order of the filter to be low. Second-order filters are common in this application while the low-order filter cannot do a very thorough job of rejecting signals at the unwanted image frequency.

In contrast, the complex image rejection filter proposed in Figure 6 is centered at a fixed intermediate frequency, and does not have to be tunable. The quality of image rejection is limited only by the ability of the filter to reject signals at negative frequencies. The ability of the receiver to reject the image of a signal is calculated by the **Image Rejection Ratio (IRR)**.

IRR, as it's shown in Eq. 15, gives the ratio of the wanted to the unwanted signal and is usually expressed in  $dB$ . Ratio of  $> 60dB$  is achievable if the system is designed well [2].



$$IRR \triangleq 20 \log_{10} \frac{A_r}{A_{r2}}$$

*Eq. 15*

In this lab, we use AM transmitter and receiver from Lab 2, and learn how to modify the receiver with a complex filter to reject the image of the signal. Then we'll calculate the IRR to determine the ability of our design in rejecting the image of the signal. The lab procedure will illustrate how to modify the AM receiver with the provided complex filter.



## Pre-Lab:

**LabVIEW Note:** In order to be able to run all the VI's and use all the LabVIEW icons, run LabVIEW 2015 (32bit) version for this lab.

A complex filter has been provided in **ChebyshevHilbert.vi** as a sub-VI. Use this sub-VI along with the TX and RX VI's built in Lab 2. The task is to use the ChebyshevHilbert VI, modify the RX and add blocks as needed to find the frequency response of the complex filter and reject the image of the signal.

## Lab Procedure

1. Connect the loopback cable and attenuator between the TX 1 and RX 2 antenna connectors. Connect the USRP to your computer with an Ethernet cable and plug in the power to the radio. At this point LED's **D** (firmware loaded) and **F** (power on) should be illuminated, as should the green light on the left side of the Ethernet connector.
2. Run LabVIEW and open the transmitter and receiver VI's that you created in Lab 2 (Amplitude Modulation).
3. Ensure the transmitter and receiver VI's are set up based on Table 1, Table 2 and Table 3.

*Table 1: Transmitter Settings*

Field	Settings
Device Name	192.168.10.2
IQ Rate	200 kS/sec
Carrier Frequency	915.1 MHz
Gain	20 dB
Active Antenna	TX1

4. Run the transmitter and receiver VI's and verify that the demodulated message appears at the receiver output.



Table 2: Transmitter Message Settings

Field	Message Settings
Start Frequency	10 Hz
Delta Frequency	100 Hz
Number of Tones	1
Modulation Index	1
Message Length	200,000 samples

Table 3: Receiver Settings

Field	Settings
Device Name	192.168.10.2
IQ Rate	1 MS/sec
Carrier Frequency	915 MHz
Gain	0 dB
Active Antenna	RX2
Number of Samples	200,000

### Setting up the Software:

#### Transmitter:

Use the AM TX you have already built in Lab 2.

#### Receiver:

1. Modify the AM RX VI to indicate the strength of the received carrier,  $A_r$ . To calculate  $A_r$ , recall that the output of an AM demodulator is:

$$r_{out}(t) = A_r \left[ 1 + \mu \frac{m(t)}{m_p(t)} \right]$$

Eq. 16



Since the message  $m(t)$  has an average value of zero, the average of  $r_{out}(t)$  will be the received carrier  $A_r$ . There are multiple ways to calculate the average of  $r_{out}(t)$ . You can use a LPF whose cutoff frequency is below the lowest frequency component of  $m(t)$ , or find a suitable signal-averaging VI. **Basic Averaged DC-RMS** icon which calculates the DC and RMS value of the input vector is a suitable VI.

- I. Place the Basic Averaged DC-RMS VI shown in Figure 7 in your VI.

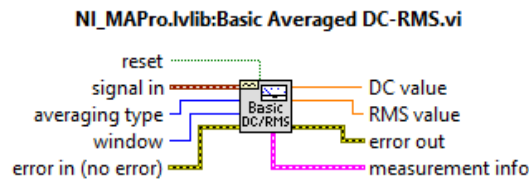


Figure 7: Basic Averaged DC-RMS VI

The data type of input **Signal in** is waveform which means we need to place a **Build Waveform** VI to generate waveform components. So:

- II. Use a Build Waveform VI to construct the waveform.
- III. Feed the Build Waveform VI with the output of the rectifier (envelope detector).

In your RX VI, you have already placed the **Get Waveform Component** VI after the **niUSRP Fetch Rx Data** to return the analog waveform components. Connect the **dt** of this VI to the **dt** of your newly added Build Waveform VI. If the Build Waveform VI doesn't have the **dt** input, simply put the mouse cursor on the block and expand it to add another output. Then click on the new output and select the **dt**:

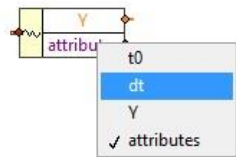


Figure 8: Build Waveform VI

- IV. Connect the output of Build Waveform VI to the signal in input of Basic Averaged DC-RMS VI.

V. Use a **Numeric Indicator** to show the carrier level (DC value) output of Basic Averaged DC-RMS VI.

2. Run the transmitter and receiver VI's and record the value of the received carrier  $A_r$  in your lab report.
3. Use Eq. 9 to determine the image frequency  $f_{IM}$ . Set the transmitter frequency to  $f_{IM}$  and then run the transmitter and receiver VI's and record the value of the received carrier  $A_{r2}$ . Compute the Image Rejection Ratio (IRR) given by Eq. 15 and record the results in your report. (1 point)

**New Receiver:**

The architecture of amplitude demodulator is shown in Figure 9. The received signal is passed through a BPF, the real part of signal is used for rectifier (envelope detector), then the LPF extract the transmitted message.

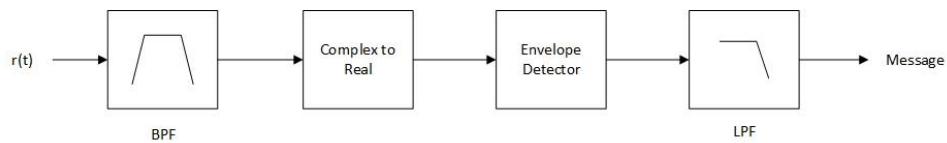


Figure 9: Amplitude Demodulator

4. Now the task is to modify the receiver to filter the image signal by using a complex filter:
  - I. In order to replace the BPF (the 5<sup>th</sup> order Chebyshev filter) with the complex filter, ChebyshevHilbert.vi, delete the current BPF and select the provided Chebyshev Hilbert sub-VI (Functions > Select a VI...). This sub-VI, shown in Figure 10, uses a Chebyshev filter and two Hilbert filters to generate a complex BPF.

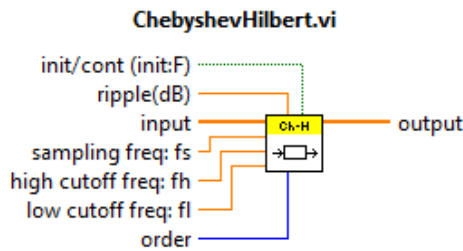


Figure 10: Chebyshev-Hilbert Icon



- II. Since we'll be working with complex values, the complex to real conversion is no longer needed, so delete the Complex to Re/Im VI.
- III. Set up your filter by adding the required constant blocks based on the Table 4:

Table 4: Chebyshev-Hilbert Filter Settings

Settings	Value
Sampling frequency	1MS/s
High cutoff frequency	105 kHz
Low cutoff frequency	95 kHz
Order	5
Ripple	Default (0.1 dB)

- 5. Save your receiver in a file whose name includes the letters “RxAMimage” and your initials. Be sure to save your transmitter as a “vi” and not a “vit” (template). **Note: You need to upload your program file with your lab report.**
- 6. Set the transmitter’s carrier frequency to  $f_c = 915.1 \text{ MHz}$ .
- 7. Run the transmitter and receiver VI’s, and measure  $A_r$ . Record the result and screenshot in your report. (1 point)
- 8. Now in order to construct the image signal, set the transmitter’s carrier frequency to  $f_{IM}$ .
- 9. Run the transmitter and receiver VI’s, and measure  $A_{r2}$ . Record the result and screenshot in your report. (1 point)
- 10. Calculate the IRR with Eq. 15 and compare the result you obtained with regular BPF. Have you rejected the image? Describe what you observe. (1 point)

**FFT Power Spectrum:**

- 11. In this section, we’d like to observe the signal and its image in the frequency domain. In order to observe the spectrum of the signal returned by the Fetch Rx Data VI, you need to use the **FFT Power Spectrum and PSD** VI. This block calculates the averaged power spectrum or power



spectrum density of time signal. The input signal is in time domain and the power spectrum output signal is shown in frequency domain. Do the following steps to set up the power spectrum correctly:

- I. Add FFT Power Spectrum and PSD VI to your Rx.

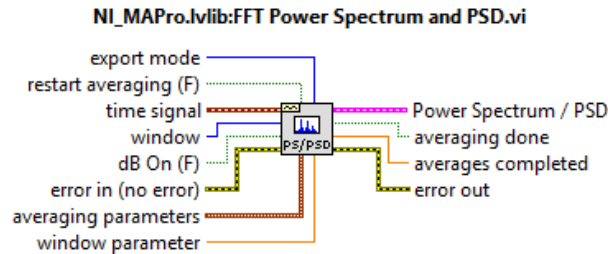


Figure 11: FFT Power Spectrum and PSD VI

- II. Wire the output of Fetch Rx Data VI to the **time signal** input.
  - III. Use a **Waveform Graph** to show the output of Power Spectrum/PSD. Note that the output wire and the Waveform block turn to pink color which means the data type is string.
  - IV. Toggle to the Front panel and change the name of the Waveform Graph to Power Spectrum.
  - V. Double click on y-axis label and change it Amplitude ( $V_{rms}^2$ ).
  - VI. Double click on x-axis label and change it to Frequency (Hz).
12. Set the transmitter's carrier frequency to  $f_c = 915.1 \text{ MHz}$ .
  13. Run the transmitter and receiver VI's, and observe the output of power spectrum. Record the result in your report. (1 point)
  14. Set the transmitter's carrier frequency to  $f_{IM}$ .
  15. Run the transmitter and receiver VI's, and observe the output of power spectrum again. Do you observe the image of signal? Describe your observation and record the result in your report. (1 point)

### Report (4 points):

The reports must be prepared in PDF or Microsoft Word format and should follow the standard of laboratory report. The report shall consist of:

1. A title page, including the name and number of the lab

2. The object, purpose and goal of the experiment shall be on the second page
3. A quick description on how the experiment is being done in 1-2 paragraphs
4. Calculations
5. Graphs
6. Pictures
7. Results
8. Conclusion
9. Tx and Rx VI files

All reports are due one week after the receiver experiment is completed and each team must submit only one report along with all the results and necessary files in a zip format file to the Moodle page.

